Hermitian Operators:-

Observable Name	Observable Symbol	Operator Symbol	Operator Operation
Position	x (r)	$\hat{X}$, $\hat{R}$	multiply by x; multiply by r
Momentum	p_x	$\hat{P}_x$	$-i\hbar \frac{\partial}{\partial x}$
	P	$\hat{P}$	$-i\hbar\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)$
Kinetic energy	K_x	$\hat{K}_x$	$\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$
	K	$\hat{K}$	$\frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$
Potential energy	$U(x)$; $U(x, y, z)$	$U(\hat{x})$; $U(\hat{x}, \hat{y}, \hat{z})$	~~My~~ multiply by $U(x)$; multiply by $U(x, y, z)$
Total energy	E	$\hat{H}$	$\frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) + U(x, y, z)$
Angular momentum	$I_x = yP_z - zP_y$; $I_y = zP_x - xP_z$; $I_z = xP_y - yP_x$	$\hat{L}_x$; $\hat{L}_y$; $\hat{L}_z$	$-i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$; $-i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$; $-i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$

All are linear operators which are quantum mechanical and with certain properties.

According to the postulate of quantum mechanical operators, in any measurement associated with observable and operator $\hat{A}$, the only values that are ever observed are Eigen values which are complex quantities, but certain eigen values may be real quantities.

$$\hat{A}\psi = a\psi \quad \text{—— (1)}$$

complex real

Quantum mechanical operators have only real values.

On multiplying eq (1) with ψ^* and integrating

$$\int \psi^* \hat{A} \psi \, dx = \int \psi^* a \psi \, dx$$

$$= a \quad \text{—— (2)}$$

Taking complex conjugate of eq (1)

$$\cancel{\hat{A}\psi} \quad \hat{A}^* \psi^* = a^* \psi^* = a\psi^* \quad \text{—— (3)}$$

where $a^* = a$, since a is ~~at~~ a real value.

Multiply eq (3) with ψ from left and integrating

$$\int \psi \hat{A}^* \psi^* dx = a \int \psi \psi^* dx = a \quad \text{—— (4)}$$

Eq. Left hands of eq. (2) and (4) we get

$$\int \psi^* \hat{A} \psi \, dx = \int \psi \hat{A}^* \psi^* \, dx \quad \text{—— (5)}$$

The operator $\hat{A}$ must satisfy the equation above to assure that its eigenvalues are real.

* Any operator that satisfies the equation above for any well-behaved function is called a <u>Hermitian</u> <u>operator</u>. In general Hermitian operators can be defined as an operator that satisfies

$$\int_{-\infty}^{\infty} f^* \hat{A} f \, dx = \int_{-\infty}^{\infty} f \hat{A}^* f^* \, dx \quad — ⑥$$

where $f(x)$ is a well behaved function. Hermitian operators have real eigenvalues.

* There are linear and Hermitian operators for the observable in quantum mechanics.

* The table in the beginning has all the operators Hermitian.

Deduce a given operator is Hermitian or not?

Consider an operator,

$$\hat{A} = \frac{d}{dx}$$

Now let us change it into the equation as

$$\int_{-\infty}^{\infty} f^* \frac{d}{dx} f \, dx = \int_{-\infty}^{\infty} f^* \frac{df}{dx} dx = \Big| f^* f \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f \frac{df^*}{dx} dx \quad — ⑦$$

For a wave function to be normalizable it must vanish at infinity, so the first term on the right hand side should be zero.

Thus we have

$$\int_{-\infty}^{\infty} f^* \frac{d}{dx} f \, dx = -\int_{-\infty}^{\infty} f \frac{d}{dx} f^* \, dx \quad \text{— (2)}$$

For an arbitrary function $f(x)$, $\frac{d}{dx}$ does not satisfy the eq. and hence it is not Hermitian.

Let us consider the momentum operator $\hat{p}_x$

$$\hat{p} = -i\hbar \frac{d}{dx}.$$

Now substituting it for Hermitian operator equation and integration

$$\int_{-\infty}^{\infty} f^* \left(-i\hbar \frac{d}{dx}\right) f \, dx = -i\hbar \int_{-\infty}^{\infty} f^* \frac{df}{dx} \, dx$$

$$+ i\hbar \int_{-\infty}^{\infty} f \frac{df^*}{dx} \, dx$$

and

$$\int_{-\infty}^{\infty} f \, \hat{p}^* f^* \, dx = \int_{-\infty}^{\infty} f \left(-i\hbar \frac{d}{dx}\right)^* f^* \, dx$$

$$= i\hbar \int_{-\infty}^{\infty} f \frac{df^*}{dx} \, dx$$

Hence the momentum operator is Hermitian.

Q: Prove Kinetic energy operator is Hermitian.

$$\boxed{\hat{K} = \frac{-\hbar^2}{2m} \frac{d^2}{2m\,dx^2}}$$

Solve on paper & send the scanned answer on

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